A Minimum Contribution Mechanism for the Provision of Public Goods

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Abstract

Public goods provision continues to be a major problem of interest for economics. Many current methods require government level intervention. In cases where governments are unable or unwilling to intervene, lower power mechanisms are required. Current best practice in this case is the provision point mechanism, typically used by Kickstarter, but this mechanism does not eliminate the potential for the free rider problem. We propose a novel minimum contribution mechanism where each player makes an offer and then each contribution is equal to the lowest offer made. This mechanism eliminates the free rider problem and implements the Lindahl (1958) equilibrium in weakly dominant strategies.

1 Intro

A great deal of effort in economics has been devoted to finding ways to efficiently provide public goods (and avoid public bads) in the presence of the free rider problem. A large portion of the literature has been devoted to finding mechanisms which allow a government or central authority with imperfect information to find the efficient amount of public good to provide.[1] This paper looks at a different problem, how can we promote the provision of public goods when no central authority exists?

Providing public goods in the absence of central authority is of substantial importance in arenas where political will for public good provision is limited or where no overarching authority exists. It can also be critically important when public goods have international impacts or when donating parties are themselves countries. This problem requires a mechanism which is individually rational and budget balanced, which most centralized mechanisms are not. Ideally, such mechanisms are also simple and do not require participants to pay more than they offer.

We propose a minimum contribution mechanism (MCM) where players make offers simultaneously and then pay out proportional to the lowest adjusted offer among all participants. This approach eliminates the free rider problem in a manner that has several appealing properties. In the first part of the paper, we described the MCM and several variants. Then we test the mechanism in a laboratory experiment. The mechanism is budget balanced, individual rational and implements the strongly efficient outcome in weakly dominant strategies.

These appealing properties come from three technical features of the mechanism. First, offers are adjusted based on Lindahl prices which allows for fair efficient outcomes.[2] Lindahl prices are hypothetical prices for public goods under which every individual pays a price for the good that is equal to their marginal benefit and where every player demands the same amount of the public good. The use of Lindahl prices help match individual marginal incentives with planner incentives to implement efficient outcomes. Second, the pinning contributions to a single pivotal player allows us to match these marginal incentives while maintaining

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[1] Clarke (1971); Groves and Loeb (1975); Myerson and Satterthwait (1983); Laffont (1987); Falkinger et al. (2000); Grüner and Koiryama (2012)

[2] Lindahl (1958)

a balanced budget. This also means the mechanism generates defined off path outcomes, which do not exist in the standard Lindahl Framework. Third, using the minimum contribution as our pivot ensures the mechanism is individually rational for all players and makes sure no one pays more than their offer. Note that the minimum contribution element provides agents with a essentially a veto power, which is essential for ensuring that the mechanism provides Pareto improvements. This point was first argued by Wicksell (1958) and more recently discussed by Van Essen and Walker (2017).

Because the MCM implements the Lindahl (1958) equilibrium, the outcome will be fair in the sense that players pay in proportional to their marginal benefit from the public good at equilibrium. Lindahl (1958) equilibrium is generally considered a very fair equilibrium.[3] However, Lindahl (1958) equilibrium can generate "unfair" seeming outcomes in scenarios where the ratio of marginal benefits from the public good are unstable (Van Essen (2021)).

The preeminent mechanism in for providing public good in the absence of government intervention is the provision point mechanism mechanism (PPM) proposed by Bagnoli and Lipman (1989). Their mechanism involves setting a contribution threshold equal to the efficient amount. If offers meet the threshold, the good is provided. Otherwise, it is not. This mechanism implements efficient provision in all undominated equilibria of the game.

Our MCM has several advantages over a standard PPM. First and foremost, in the MCM the efficient outcome is implemented as the result of each player choosing their unique weakly dominant strategy. This is potentially an important contribution, since threshold-based mechanisms often lead to coordination problems and games of chicken where players do not contribute in the hopes of getting their more preferred equilibria (ones where they do not pay as much).

This also means our MCM is to some degree lower risk for both policy makers and participants as there is less uncertainty about outcomes. In a standard public goods game players face a risk that they will be the only contributor and a risk that some players will not contribute. PPM strongly mitigate the first risk but due not eliminate the second. A MCM removes both of these free riding risks.

Both the provision point mechanism and the MCM are generally information intensive. Setting the correct provision point requires knowing player preferences. Setting the correct offer multipliers in the MCM similarly requires such preferences to be known by the mechanism designer. Both the PPM and the MCM require less information if come structure is imposed. The PPM does not require knowledge of preferences if the public good is binary.[4] The MCM does not require knowledge about the overall marginal return on the public good as long as the players receive a known constant share of the marginal benefits.

Traditionally, The Wilson (1987) critique has been leveled at mechanisms of this type due to their reliance on the knowledge of preferences. If preferences are commonly known, why do the agents not simply contract on an efficient outcome? We counter that the act of negotiating a contract is often a source of barriers and frictions.[5] Substantial resources can be spent in trying to secure more of the surplus generated by a contract and sometimes parties fail to agree even on highly profitable joint ventures. Unlike with take it or leave it offers, trying to restart or rerun the MCM will not lead to different results.

There are also situations where contracting is effectively impossible due to legal or practical barriers. Furthermore, PPMs are frequently used by real world charities[6] and form the basis of funding platforms like Kickstarter, suggesting mechanisms of this type are worth considering.

It is likely that MCMs will be particularly useful in scenarios where the ratio of benefit from the public good easy to approximate and relatively stable. For example, if the public good is a research project conducted by several manufacturing firms in the same industry to help reduce the cost of production per unit, the benefit from the research is likely to be proportional to the market share of each firm.

The fact that the MCMs do not require efficient contribution levels as an input may also make them more resistant to manipulation. Anecdotally, in many public goods and bads problems in the past, individuals

[3] Sato (1987); Buchholz and Peters (2007)

[4] Bagnoli and Lipman (1989). Note that a low information sequential PPM mechanism can theoretically be used in cases where better resolution is needed, but adding more provision levels has been shown to hinder performance in the lab. See Bagnoli et al. (1992) and Normann and Rau (2014).

[5] There is a large literature on the topic of frictional and incomplete contracts. A small sample of the related papers includes Antràs (2003); Antràs (2005); Acemoglu et al. (2007).

[6] Bagnoli and McKee (1991)

who would be expected to bear a large portion of the costs have typically attempted to muddy the waters on how much public good or bad is desirable and how much each individual should pay. Consider the behavior of petroleum companies regarding global warming. The MCM shuts down this avenue of manipulation.[7] Also, in settings where the public good is provided by an organization, there may be an incentive to increase contributions beyond efficient levels. With a MCM, there is no way to manipulate values in order to get over-provision of the public good.

As alluded to, our MCM can be adapted to address questions of public bads and resource extraction, which is difficult to do with the PPM due to lack of enforceable exclusion.[8]

The provision point mechanism has been experimentally shown to provide generally provide efficient outcomes in scenarios where groups are given time to converge to an equilibrium or when the focal contribution levels generate an efficient outcome, but there is still room for improvement in more hostile environments.[9] We hope our MCM might help fill in the gaps and perform well with single shot games without focal equilibria.

There is also evidence that a lot of observed public good contributions in the lab are the result of confusion or a poor understanding of public-private trade-offs inherent in the game.[10] When players are unsure of the rules or incentives in an economic game, they are likely to choose actions randomly or pick actions in the middle of the range to avoid large errors. The public-private trade-off is still a feature of provision point mechanisms. It is therefore possible that the PPM's success in the lab may be partially due to confusion. It may be less successful with sophisticated actors like firms, professionals, and nations or in conditions where default behavior is not contributing. On the other hand, the MCM eliminates the public-private trade-off, so sophistication and defaults are unlikely to present a problem.

Our experiment is designed to see whether the MCM performs well in scenarios where the PPM does poorly. Games will be one shot with no feedback and in half of treatments there will be no obviously correct way for all players to contribute in order to reach efficiency. Instructions will be written with the lessons of Ferraro and Vossler (2010) in mind, presenting a range of example payoffs that clearly show the public-private tradeoff as well as a specialized calculator for each problem. The agents will be individual college students, however, so lack of sophistication may still be an issue. Since the PPM performs in the presence of heterogeneity and homogeneity[11] we will include treatments with these properties in order to see if the MCM is as successful in both contexts.

The paper is laid out as follows. Section 2 lays out the basic framework. Section 3 describes the MCM and its properties as well as introducing a dynamic version of the game for when the mechanism maker does not know prices. Section 4 provides a background for how the mechanism works in imperfect conditions including the presence of noise and manipulation. This includes a method for comparing the outcomes of various imperfect mechanisms including the MCM and various distortions of the VCM.

Section 6 lays out an outline for an experiment testing the theory. More to come in the future.

1.1 Literature

Buchholz et al. (2014) consider a mechanisms for the private provision of public good which is based on social norms and punishment. This type of mechanism requires additional structure on the game and preferences including pre-commitment of enforcement resources. Also, the Buchholz et al. (2014) mechanism in general does not produce an efficient outcome even disregarding the resources spent on enforcement.

The closest thing in the literature to the MCM we use was proposed as an auction mechanism by Goeree et al. (2005). They examined games where an item was auctioned off to raise money for a linear public good. Goeree et al. (2005) found that the minimum price all pay auction was the revenue maximizing auction. Our models intersect in a special case of each. The special case of the Goeree et al. (2005) model is the one in which the value of the auctioned good is zero. The special case of our model is one in which the public goods provision technology is linear and the impact is homogeneous.

[7] Other avenues of manipulation may still exist. See Section 4.2.3 for more discussion of manipulation in the MCM

[8] See Appendix B

[9] Bagnoli and McKee (1991), Rondeau et al. (1999)

[10] Ferraro and Vossler (2010)

[11] Rondeau et al. (1999)

Empirically, a mechanism like the one proposed by Goeree et al. (2005) did not perform well when tested.[12] It failed to reach theoretical contribution levels and failed to beat other public good funding mechanisms. We hypothesize that the auction framing may have effectively reintroduced the free rider problem psychologically even while the minimum price component eliminated the free rider problem mathematically. Individuals are often reluctant to contribute to public goods, since they do not want to subsidize free riders. Similarly, individuals are likely to be reluctant to bid in all pay auctions, because they do not want to subsidize the single auction winner. In both cases, participants do not want to risk footing the bill for someone else's unfair/unearned benefit.

Experiments have tested public good provision in many different settings and with many different mechanisms. Ledyard (1995) provides a good survey of some of the earlier work in this area showing that contributions tend to be responsive to communication, group size, and the marginal impact of the public good.

Carpenter (2007) found that group size does not decrease public good contributions when punishment is possible and contributions can be monitored. They also provide a good review of other experiments with punishment in public goods games. More recently, Stagnaro et al. (2017) find that centralizing punishment mechanisms increase pro-social behavior without increasing actual amounts of punishment.

Corazzini et al. (2010) and Lange et al. (2007) both test a number of fundraising mechanisms for public goods including voluntary contribution mechanisms, charity auctions, and charity lotteries. Both studies found that lotteries performed best out of all tested mechanisms in terms of maximizing contributions. Carpenter and Matthews (2017) tested types of charity raffles and found that, contrary to theoretical predictions, a "pay what you want" raffle works best.

Reif et al. (2017) test the mechanism proposed by Buchholz et al. (2014) and find that it sustains substantial public good provision but can lead to over-investment in norms enforcement.

2 Theory

In this game there are players $i \in \mathcal{I} = \{1, ..., I\}$. All players make offers $\boldsymbol{x} = (x_1, ..., x_i, ..., x_I)$ simultaneously where $x_i \in \mathbb{R}^+$. Based on the offers made, a set of contributions is generated $\boldsymbol{y} = (y_1, ..., y_i, ..., y_I)$. The mapping from offers to contributions depends on the mechanism involved.

Individuals get utility from the total public good provided and private consumption in the form, $u_i(Y, -y_i)$, where $u_i(\bullet)$ is a weakly concave, increasing, and satisfies the Inada like conditions $\frac{d}{dy_i}u_i(0,0) = 0$, $\frac{d}{dY}u_i(0,0) > 0$. Here $Y = \sum_{i=1}^{I} y_i$. For the main body of the text, we assume $u_i(\bullet)$ is everywhere continuously differentiable, but this is not a critical assumption. For convenience we normalize $u_i(0,0) = 0$.

In terms of solutions, we will be focusing on pure strategy Nash equilibria and equilibrium in weakly dominant strategies when it is available.

2.1 Efficiency

In this setup, a Pareto Efficient allocation is a set of contributions y_i summing to Y for which there is no other allocation which provides all players at least as high of payoffs and at least one player higher payoffs. Samuelson (1954) showed that an allocation in a public goods economy is Pareto Efficient, if and only if

$$\sum_{i=1}^{I} MRS_i(Y, y_i) = 1$$

Where the marginal rate of substitution

$$MRS_i(Y, y_i) := \frac{\frac{d}{dY}u_i(Y, -y_i)}{\frac{d}{dy_i}u_i(Y, -y_i)}$$

[12]Corazzini et al. (2010)

2.2 The VCM

As a point of reference, we first consider the public goods contribution problem when no mechanisms are in place. The amount players offer is equal to the amount they pay in to the public good. In this game, each player is picking

$$y_i^{V*} = \arg\max_{y_i} u_i \left(\sum_{j=1}^{I} y_j, -y_i \right)$$

With a total equilibrium contribution level $Y_V^* = \sum_{j=1}^{I} y_i^{V*}$

Under our assumptions, this solution satisfies the condition

$$MRS_i \left(Y_V^*, y_i^{V*} \right) = 1$$

Which means the Samuelson condition is generally not satisfied in this case.

2.3 Lindahl Equilibrium

In our simplified public goods economy. A Lindahl Equilibrium is a set of contributions y_i^* summing to Y^* and a set of prices p_i^* such that

$$\arg\max_{Y_i} u_i(Y_i, -p_i^* Y_i) = Y_i^* = Y^* \forall i$$

Note, that here

$$y_i = p_i^* Y_i$$

In other words it is a set of personal prices for the public good in which everyone demands the same amount of public good and the amount provided is equal to the amount demanded. Foley (1970) has shown that under the conditions we employ, the Lindahl Equilibrium exists and is Pareto Efficient. Efficiency can be seen from the optimality condition

$$MRS_i(Y^*, y_i^*) = p_i^*$$

and the fact that budget guarantees

$$\sum_{i=1}^{I} p_i^* = 1$$

Together these two facts give us Samuelson's Condition. However, Lindahl equilibrium is not easily implementable in a game theoretic sense, since there is no mechanism defining what happens outside of equilibrium. This is one of the major issues the MCM is designed to solve.

There is not generally a closed form solution for the Lindahl Equilibrium, so finding the Lindahl prices generally requires both the full knowledge of all player's preferences and significant computation. Generally knowing the preferences is a more economically significant than computing the prices except in cases with very unsophisticated agents. In Section 4, we discuss several potentially realistic cases where the knowledge and computational burden are much lower.

3 The MCM

Now we formally introduce the MCM for converting offers into contributions. This mechanism states that

$$y_i = p_i * \min_j \tfrac{x_j}{p_j}$$

Where p_i is the weight for player i and $\sum_{i=1}^{I} p_i$. Note that in this case

$$Y = \sum_{i=1}^{I} y_i = \min_j \tfrac{x_j}{p_j} \sum_{i=1}^{I} p_i = \min_j \tfrac{x_j}{p_j}$$

Under this mechanism, players pay out a weighted amount based on the lowest adjusted offer. In the case were p_i is identical across individuals, this corresponds with each player contributing the minimum offered amount.

Note that in this setup the player i picks x_i to solve

$$x_i^* = \arg\max_{x_i} u_i \left(\min \left(\tfrac{x_i}{p_i}, \tfrac{x_k}{p_k} \right), - \min \left(x_i, p_i \tfrac{x_k}{p_k} \right) \right)$$

where

$$k = \arg\min_{j \neq i} \tfrac{x_j}{p_j}$$

3.1 Properties of the MCM

The first thing to note is that under the MCM, the individual's offer selection problem can be rewritten as a problem where the individual selects a total contribution level.

$$Y_i^* = \arg\max_{Y_i} u_i \left(\min \left(Y_i, Y_{min}^i \right), -p_i \min \left(Y_i, Y_{min}^i \right) \right) \tag{1}$$

Where

$$Y_{min}^i = \min_{j \neq i} Y_j$$

Note that $\min \left(x_i, p_i \frac{x_k}{p_k} \right) = p_i * \min \left(\frac{x_i}{p_i}, \frac{x_k}{p_k} \right)$.

There are a few things which are essentially immediate from this formulation.

Proposition 1. *The MCM with Lindahl weights implements the corresponding Lindahl Equilibrium in weakly dominant strategies.*

Proof. Note that 1 is a restricted version of

$$Y_i^* = \arg\max_{Y_i} u_i(Y_i, -p_i^* Y_i)$$

The solution of this is $Y_i^* = Y^*$. Note that, since $u_i(\bullet)$ is concave, the objective function is weakly increasing on the interval $[0, \max(Y^*)]$.

By definition, the objective function is the same for all elements of Y^* and is worse for any $Y_i > \max(Y^*)$. Therefore, regardless of Y_{min}^i it is weakly optimal to pick $Y_i \in Y^*$ and the player is ambivalent between all elements of the set.

□

This is the most important feature of the mechanism. It eliminates the free rider problem by refunding those who make high offers. It is important to note that, while the MCM does implement the efficient outcome as the result of each player choosing their unique weakly dominant action, there are still other equilibria.

Corollary 1. *The equilibrium of the minimum contribution game is not generally unique*

Proof. Simply note that, in addition to the previous equilibrium of $Y_i \in Y^* \forall i$ there is another equilibrium where $Y_i = 0 \forall i$. Also Y^* may not be single valued, so there may be multiple efficient equilibria.
□

However, these other equilibria are not efficient and are not the result of dominant strategies, so they are generally less plausible.

In addition to implementing the efficient outcome, this mechanism also has a number of nice features. Firstly, it is budget balanced. The amount spent on the public good is the exact sum of contributions. This is very helpful for mechanisms that can't rely on governments or other entities which can destroy and create money. In addition, no player will ever pay out more than their offer, because

$$p_i * \min_j \frac{x_j}{p_j} \leq x_i$$

As a result, the mechanism could be implemented through a system of refunds.

Second, the mechanism is individually rational in the sense that a rational player (one choosing $Y_i \in [0, \max(Y^*)]$) will always make weakly more than 0 regardless of how others play. This mechanism always generates Pareto improvements with optimizing agents due to the veto power held by individual contributors.

Finally, the mechanism has a certain type of fairness in scenarios where some individuals get consistently higher marginal benefits from public goods. An individual who gets higher marginal benefit from the public good pays proportionally higher cost.

In many cases the mechanism maker and participants may not have common knowledge of all player's preferences. If only the mechanism maker is unaware, efficiency is still achievable with only minor sacrifices (see Appendix C). In cases where preferences are not common knowledge, it may be possible to extract a player's information about the incentives of others. However, this extraction will be incomplete, since information about the preferences of others will often contain information about one's own preferences. To see one example where information is extracted efficiently see Section C.1.

In many cases it will not be possible to fully extract information from players. We discuss the impacts of such problems in the next section.

4 Special Cases and Imperfect Implementation

There are a number of special cases where the Lindahl prices have convenient forms. These environments are very useful for examining issues like manipulation and imperfect information and for comparing results.

As noted in the introduction, this mechanism is not designed to elicit information about the preferences of the participants. Instead this mechanism is designed to solve a coordination problem in the absence of overarching government authority. As such, the mechanism is information intensive, requiring the $u_i(\bullet)$s to be commonly known or at least known to the mechanism facilitator. Imperfect information can reduce the effectiveness of the mechanism.

There is also a practical question of how, even if the $u_i(\bullet)$s are known to participant, are they assigned in the mechanism. If the mechanism is implemented by a neutral third party, this should not be a problem. However, it may be difficult to find a fully informed neutral third party in practice. In cases where the mechanism is implemented by an interested party there is the potential for manipulation by the mechanism maker. We consider both of these problems below with the aid of some additional structure.

4.1 Quasi Linear Preferences

Consider an environment with quasi-linear preferences so

$$u_i(Y, y_i) = g_i(Y) - y_i$$

Where $g_i(\bullet)$ is increasing, concave, and continuously differentiable. In this setting we can make stronger statements about efficiency, because all Pareto optimal allocations are also strongly efficient in the sense of maximizing the sum of utilities. With quasi-linear utility, we can meaningfully talk about a social planner who wants to maximize social utility.

In other words they want to choose

$$Y^* = \arg\max_Y \sum_i (g_i(Y)) - Y \tag{2}$$

Because the payoffs are linear in contributions, the planner does not care how the contributions are divided among participants. The solution to the planner problem satisfies the FOC

$$\sum_i (g_i'(Y^*)) = 1$$

Since the marginal cost of contribution is always 1, the marginal rate of substitution for a player is then

$$MRS_i(Y) = g_i'(Y)$$

Note that, since the Lindahl Equilibrium is Pareto efficient, and Pareto efficiency coincides with strong efficiency in when utility is quasi-linear, the Lindahl Equilibrium is strongly efficient. Together these facts mean it is computationally quite easy to find the Lindahl prices in this case.

$$p_i^* = g_i'(Y^*)$$

Instead of a complex multi-dimensional optimization problem over prices, one merely needs to solve the single dimensional planner problem and plug in the result.

4.2 Proportional Benefit

In order to address questions of manipulation and imperfect information in a disciplined way, we need to impose still more structure on preferences. In addition to quasi-linearity we require that each individual receive a constant proportion of the benefit from the public good. In other words we require

$$u_i(Y, y_i) = \delta_i g(Y) - y_i$$

Where $\sum \delta_i = 1$ and $g(\bullet)$ is increasing and concave. We drop the assumption of continuous differentiability here, because it does not substantially simplify the discussion. In this environment the planner problem becomes

$$Y^* = \arg\max_Y \sum_i (g_i(Y)) - Y \tag{3}$$

and the Lindahl Prices are simply

$$p_i^* = \delta_i$$

While this structure is quite strong, we contend that it is at least approximately valid for many real world settings. In the Introduction we mention an R&D example where benefit was proportional to market share. In general, there are a number of situations where the benefit from a public good is roughly proportional to obvious features of an entity. Ocean conservation may provide benefits proportional to the size of a country's fishing industry. The benefits of a parking lot expansion in a mall may be roughly proportional to a store's peak-time customer load. The benefits of a disease cure to one nation are roughly proportional to the average yearly cost of dealing with the disease in that country.

We will want some easy way to compare outcomes of the MCM to the VCM in the presence of manipulation and imperfect information. Given the proportional benefit structure of this game, we can make use of the following proposition to easily compare outcomes.

Proposition 2. *If $Y(k) = \arg\max_Y \kappa * g(Y) - Y$, then $Y(k)$ is increasing in k (in the strong set order sense)*

Proof. This result is an application of Milgrom and Shannon (1994). We simply need to show that $f(Y,k) = k * g(Y) - Y$ has increasing differences in (Y,k). To see this, consider $Y > Y'$. We know $f(Y,k) - f(Y',k) = k * (g(Y) - g(Y')) - (Y - Y')$. Since $g(\bullet)$ is increasing in Y, this difference is increasing in k.
□

Note that we will not be theoretically comparing the performance of the MCM and the provision point mechanism under imperfect information, because the distortions affecting these two mechanisms are essentially orthogonal. The MCM is impacted by distortions in δ_i but not $g(Y)$ while the provision point mechanism is impacted by distortions in $g(Y)$ but not δ_i. The results are therefore largely mechanical and come down to determining which type of distortion is larger given assumptions.

4.2.1 VCM Game with Proportional Benefit

As a point of reference, we first consider the public goods contribution problem when no mechanisms are in place. The amount players offer is equal to the amount they pay in to the public good. In this VCM game, each player is picking

$$y_i^* = \arg\max_{y_i} g_i\left(\sum_{j=1}^{I} y_j\right) - y_i$$

This differs substantially from the planner problem. In order to be compatible with Proposition 2, we employ the following result.

Proposition 3. *In all pure strategy Nash equilibria, the VCM implements some*

$$Y_V^* \in \arg\max_Y \left(\max_i (\delta_i) * g(Y) - Y\right) \tag{4}$$

Proof. Note, by the concavity of $g(Y)$, the argmax must be an interval. Proof is by contradiction. Imagine that there is a pure strategy Nash equilibrium where a $Y < \min(Y_V^*)$ was implemented. Say

$$i^* \in \max_i (\delta_i)$$

Player i^*has an objective function of

$$\max_{y_{i^*}} \delta_{i^*} g(Y_{-i^*} + y_{i^*}) - y_{i^*}$$

Where $Y_{-i^*} < Y_N^*$ is the sum of contributions from players other than i^*. This objective function can be rewritten as a total player contribution selection problem

$$\max_Y \delta_{i^*} g(Y) - Y + Y_{-i^*}$$

$$st\, Y \geq Y_{-i^*}$$

Since Y_{-i^*} is fixed, this is the same problem as the one in the Proposition with the added restriction. Note that the restriction cannot bind at the optimum if we are assuming a $Y < \min(Y_V^*)$ was implemented. As such, if $Y < Y_V^*$ is implemented, player i^*has a profitable deviation by increasing their contribution.

Now imagine there is a Nash equilibrium which implements Y higher than $\max(Y_V^*)$. Let $\hat{i}$ be some player that contributes in this equilibrium. They have an objective function

$$\max_{y_{\hat{i}}} \delta_{\hat{i}} g(Y_{-\hat{i}} + y_{\hat{i}}) - y_{\hat{i}}$$

Define

$$\hat{Y}^* \in \arg\max_Y \delta_{\hat{i}} g(Y) - Y$$

Note Player $\hat{i}$'s objective function is strictly decreasing in Y for all $Y > \max\left(\hat{Y}^*\right)$

Player $\hat{i}$'s objective function can be rewritten as a total player contribution selection problem

$$\max_Y \delta_{\hat{i}} g(Y) - Y + Y_{-\hat{i}}$$

$$st\, Y \geq Y_{-\hat{i}}$$

Which has a solution of a $\max(\hat{Y}^*, Y_{-\hat{i}})$, which by Proposition 2 is less than the implemented value, so Player $\delta_{\hat{i}}$ can profitably decrease his contribution.

Finally, we must show that there is a PSNE implementing Y_V^*. To see this note that there are PSNE's where only i^* contributes, and in such cases his optimization problem is equivalent to 4.

□

4.2.2 Incorrect Prices

First we consider what happens when mechanism is implemented imperfectly in a general sense. We will later apply more structure to make stronger statements. In this case the true value extracted by the participants is not the same value that is used in the mechanism. Say that the mechanism assigns a potentially incorrect weight p_i to each participant.

In this case each participant is choosing

$$Y_{inc}^i \in \arg\max_{Y_i} \delta_i * g\left(\min\left(Y_i, Y_{min}^i\right)\right) - p_i * \min\left(Y_i, Y_{min}^i\right) \tag{5}$$

Where

$$Y_{min}^i = \min_{j \neq i} Y_j$$

Proposition 4. *The game of incorrect prices can implement in dominant strategies the solution to*

$$Y_{inc}^* \in \arg\max_Y \left(\min_i \left(\frac{\delta_i}{p_i}\right) g\left(Y\right) - Y\right)$$

Proof. To see this, note that 5 is the same as

$$Y_{inc}^i \in \arg\max_{Y_i} \frac{\delta_i}{p_i} * g\left(\min\left(Y_i, Y_{min}^i\right)\right) - \min\left(Y_i, Y_{min}^i\right)$$

Similar to the proof of Proposition 1, the concavity of $g(\bullet)$ guarantees that player i's objective function is increasing in $Y_i \forall Y_i \in [0, \max\left(Y_{inc}^{i*}\right)]$ where

$$Y_{inc}^{i*} \in \arg\max_{Y_i} \left(\frac{\delta_i}{p_i} g\left(Y_i\right) - Y_i\right)$$

As such, it is a weakly dominant strategy to pick $Y_i \in Y_{inc}^{i*}$. If all players do this the minimum Y_i chosen gets implemented. From proposition 2, we know that the Y_{inc}^{i*}s are related by the strong set order with the lowest set corresponding to the minimum $\frac{\delta_i}{p_i}$. Therefore, the implemented Y_i must be an element of the lowest Y_{inc}^{i*} corresponding with the lowest $\frac{\delta_i}{p_i}$.

□

4.2.3 Manipulation

Say that one agent, j assigns p_i for all players with the restriction that $\sum_{i=1}^{I} p_i = 1$. He knows the true p_is, but gains some benefit from the public good and therefore is potentially on the hook for some of the cost. The game operates in two stages. First, the manipulator chooses δ_is, then all players (including the manipulator) make their offers simultaneously. Assume that after weights are assigned, players play the equilibrium described in Proposition 4.

Then there are two potentially competing pulls on the manipulator. By changing the values of p_i, they can reduce the fraction of the public good they have to pay for, but any manipulation will also reduce the total amount of public good provided, since manipulation will always reduce $\min_i \left(\frac{\delta_i}{p_i}\right)$.

The result is given by the following proposition

Proposition 5. *If the mechanism weights are determined by individual j, the MCM will implement Y^*_{man} total contributions in weakly dominant strategies, where*

$$Y^*_{man} = \arg\max_{Y} \left(\frac{1-\delta_j}{1-p_j} g(Y) - Y\right)$$

and $p_j \in [0, \delta_j]$

Proof. The manipulator will always want to pick $p_j \leq \delta_j$. We show this by contradiction. Note that he can implement Y^* by setting $p_j = \delta_j$. Say the manipulator picks $p_j > \delta_j$. Define

$$\tilde{Y}_j = \arg\max_{Y_i} \left(\delta_j * g(Y_j) - p_j Y_j\right)$$

We know

$$\delta_j * g(Y^*) - \delta_j Y^* \geq \delta_j * g\left(\tilde{Y}_j\right) - \delta_j \tilde{Y}_j > \delta_j * g\left(\tilde{Y}_j\right) - p_j \tilde{Y}_j$$

Hence p_j cannot be optimal.

Given his own mechanism weight $p_j > \delta_j$, the manipulator will want to maximize total contributions Y. To see this, note that

$$\frac{\delta_j}{p_j} * g(Y) - Y$$

is increasing in Y for all $Y \in [0, \bar{Y}^*_j]$ where

$$\bar{Y}^*_j \in \arg\max_{Y} \frac{\delta_j}{p_j} * g(Y) - Y$$

Note that $\bar{Y}^*_j > Y^*$ by Proposition 2. Therefore, the manipulator's objective function is increasing in Y for the entire range of Ys that is feasible given $p_j > \delta_j$.

Since the equilibrium Y depends on the lowest $\frac{\delta_i}{p_i}$, it is optimal to allocate the p_i in such a manner as equalizes this across all other individuals.

We know

$$1 - p_j = \sum_{i \neq j} p_i$$

So we have

$$p_i = \frac{1-p_j}{1-\delta_j} \delta_i$$

Which satisfies the summand and the requirement that $\frac{p_j}{\delta_j}$ be equalized.

Therefore,

$$\frac{\delta_i}{p_i} = \frac{1-\delta_j}{1-p_j} \forall i \neq j$$

Therefore all players other than j will make offers that implement $Y \in Y^*_{man}$, and player j will make an offer weakly greater than Y^*_{man}.

□

4.3 Comparison of Outcomes

We would now like to compare the outcomes in the manipulation and imperfect information cases with the efficient and VCM outcomes. We can use Proposition 2 for this comparison, since most of the discussed outcomes implement a total contribution that can be written as the result of an optimization of the proper form. Now we can compare different scenarios and mechanisms based on the corresponding Proposition 2 κ value in order to rank them for total public goods contributions. The following table summarizes the result

Mechanism/Situation	κ	Implemented Y
Social Planner	1	Y^*
Minimum Contribution Perfect Conditions	1	Y^*
VCM	$\max_i \delta_i$	Y^*_V
Minimum Contribution Incorrect Information	$\min_i \left(\frac{\delta_i}{p_i}\right)$	Y^*_{inc}
Homogenous Price MCM	$\min_i (I\delta_i)$	Y^*_{hom}
Minimum Contribution Manipulation	$\frac{1-\delta_j}{1-p_j}$	Y^*_{man}

Note that in all cases $\kappa \le 1$, so there is no risk of inefficient over-contribution. We know that Y^* is optimal and higher than the other implemented Ys in the table, but it is not immediately obvious how Y^*_N, Y^*_{inc}, and Y^*_{man} rank. The next few sections are devoted to exploring that ranking.

In general the MCM with manipulation will be more efficient than the VCM.

Corollary 2. *The VCM can only be more efficient than the MCM with manipulation if the manipulator's* $\delta_j > 0.5$

Proof. Since $p_j \in [0, \delta_j]$, we know that $\frac{1-\delta_j}{1-p_j} \in [1-\delta_j, 1]$. This implies that manipulation has a higher k as long as $\max_i (\delta_i) \le 1 - \delta_j$ where j is the manipulating agent.

It is only possible for the manipulation mechanisms k to be lower if $\max_i (\delta_i) > 1 - \delta_j$. Note

$$1 - \delta_j = \sum_{i \neq j} \delta_i \ge \delta_i \forall i \neq j$$

So this can only happen if $\delta_j = \max_i(\delta_i)$ in which case we need

$$\delta_j > 1/2$$

□

Note that $\delta_j > 1/2$ does not guarantee that the MCM with manipulation performs worse than the VCM, but we can construct examples where it does. See Appendix A for such an example.Note that the best case manipulators will always perform better than the VCM since $\min_j \delta_j < 1/2$

The VCM and the MCM with imperfect information cannot be easily compared without imposing a bit more structure on the δ_is and p_is. By imposing some structure we can also get insights regarding how we should expect the different mechanisms to perform as the size of the group gets large. We begin by imposing structure in the δ_is

4.3.1 Random δs and Large Groups

Say that there is random heterogeneity in the incidence of the public good of the form

$$\delta_i = \frac{\eta_i}{\sum_l \eta_l}$$

Where η_is are drawn independently from a distribution with a weakly positive domain and a CDF $F_\eta(\bullet)$ and a mean $\bar{\eta}$.

Under this assumption we immediately have a few results

Proposition 6. *If $g(\bullet)$ is continuous and $\lim_{I\to\infty} F_\eta(Ic)^I = 1 \forall c$, then as $I \to \infty$, Y_V^* converges in probability to 0.*

Proof. If $g(\bullet)$ is continuous, then by the theorem of the maximum $Y(k)$ is a continuous function of k. This means that $Y_k \to_p 0$ as $k \to_p 0$. As such, it suffices to show that $\max_i \delta_i \to_p 0$

$$\max_i \delta_i = \frac{\max_i \eta_i}{\sum_l \eta_l} = \frac{\frac{1}{I}\max_i \eta_i}{\frac{1}{I}\sum_l \eta_l}$$

Note $\frac{1}{I}\sum_l \eta_l \to_p \bar{\eta}$. Since Slutzki's Theorem guarantees that the limit of the ratio is the ratio of the limits, we just need to show

$$\frac{1}{I}\max_i \eta_i \to_p 0$$

Note that $P\left(\frac{1}{I}\max_i \eta_i < c\right) = P\left(\max_i \eta_i < cI\right) = F(cI)^I$, so by the definition of convergence in probability, we are done

□

Note that the condition on $F(\bullet)$ holds for many standard distributions (like the exponential), but it fails for extremely fat-tailed distributions like the power law distribution. In order for the VCM not to converge to zero contributions, the distribution must have a large enough tail that the largest draw as a fraction of the total draws does not tend to zero. In other words the largest proportional individual stake in the public good must remain positive.

Note that as a corollary, the MCM with manipulation can generally be expected to perform well with large groups when δs are random.

Corollary 3. *If $g(\bullet)$ is continuous and $\lim_{I\to\infty} F_\eta(Ic)^I = 1 \forall c$, then as $I \to \infty$ the implemented Y_{man}^* converges in probability to Y^*.*

Proof. To see this note that the worst case of $\bar{\delta}_j = 0$ generates a $k = 1 - \delta_j = 1 - \frac{\eta_j}{\sum_l \eta_l}$. Therefore k is bounded below by $1 - \max_i \left(\frac{\eta_i}{\sum_l \eta_l}\right)$. As discussed in the proof of Proposition 6, $\max_i \left(\frac{\eta_i}{\sum_l \eta_l}\right) \to_p 0$ under the conditions provided on $F_\eta(\bullet)$ and $Y_{man} \to_p Y^*$ as long as $g(\bullet)$ is continuous and $k \to_p 1$.

□

Note that this proposition gives worst case assuming the worst manipulator. Generally performance will be better.

4.3.2 Random Noise and Imperfect Information

In order to compare the VCM to the imperfect information and manipulator cases, we must impose some structure on the noise. We assume

$$p_i = \frac{\delta_i \epsilon_i}{\sum_l \delta_l \epsilon_l}$$

Where ϵ_is have a are drawn independently from a distribution with a weakly positive domain and a CDF $F_\epsilon(\bullet)$ and a mean $\bar{\epsilon}$. Assume δ_is are randomly generated as before independently of the ϵ_is.

Proposition 7. *If ϵ_i is not bounded above, then as $I \to \infty$ the provision level Y^*_{imp} converges in probability to 0.*

Proof. In this case, the corresponding k is given by

$$\min_i \left(\frac{\sum_l \delta_l \epsilon_l}{\epsilon_i} \right) = \frac{\sum_l \delta_l \epsilon_l}{\max_i \epsilon_i}$$

If we apply the random generation of δ_is as before, we get

$$\frac{\sum_l \eta_l \epsilon_l}{\max_i(\epsilon_i) * \sum_l \eta_l}$$

By the law of large numbers, the independence of ϵ and η, and the law of total expectation we have

$$\frac{\frac{1}{I}\sum_l \eta_l \epsilon_l}{\max_i(\epsilon_i) * \frac{1}{I}\sum_l \eta_l} \to \frac{\bar{\eta}\bar{\epsilon}}{\bar{\eta}} * \lim_{I \to \infty} \left(\frac{1}{\max_i(\epsilon_i)} \right)$$

Which simplifies to

$$\lim_{I \to \infty} \left(\frac{\bar{\epsilon}}{\max_i(\epsilon_i)} \right)$$

Which goes to 0 if $\max_i (\epsilon_i)$ is not bounded
□

Neither the VCM nor MCM with noise perform well with large groups. There are scenarios where both mechanisms converge to zero provision as the group size gets larger.

For finite group size, we can directly compare the corresponding k values to see when one mechanism should outperform the other. The MCM with noise performs better when

$$\frac{\sum_l \eta_l \epsilon_l}{\max_i(\epsilon_i) * \sum_l \eta_l} \geq \frac{\max_i \eta_i}{\sum_l \eta_l}$$

or when

$$\sum \eta_l \epsilon_l \geq \max_i (\eta_i) \max_i (\epsilon_i)$$

Notably the MCM performs better when noise and/or heterogeneity is small. It is only when both heterogeneity and noise are large that the VCM can perform better. With a sample size of one, both mechanisms are equal. As group size grows, the left hand side of the inequality grows linearly. For the right hand side to grow that fast requires highly fat tailed distributions for both η and ϵ.

5 Mechanisms for Charitable Contributions

[Incomplete]

6 Experimental Design

Incomplete

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A Manipulation Worse than VCM Example

Consider the following setup. There are two individuals with $\delta_1 = \frac{3}{4}$ and $\delta_2 = \frac{1}{4}$. Assume $g(y)$ is piece-wise linear. The first section has a horizontal length Y_1 and has a slope $4 + \epsilon$. The second piece has a horizontal length Y_2 and has slope $\frac{4}{3} + \epsilon$. The remainder has slope of the second section times δ_1 is greater than 1.

If player 1 is the manipulator, he can implement Y_1choosing $\bar{\delta}_j = 0$ or $Y_1 + Y_2$ by choosing $\bar{\delta}_j$ such that

$$\tfrac{1/4}{1-\bar{\delta}_j}(4/3 + \epsilon) > 1$$

$$(1/3 + \epsilon/4) > 1 - \bar{\delta}_j$$

$$\bar{\delta}_j > 2/3 - \epsilon/4$$

We can make Y_2 arbitrarily small such that the utility gains going from the first kink to the second are negligible, but the difference in cost is approximately $(2/3)Y_1$. If Y_1 is relatively large, manipulator will implement Y_1 which is slightly smaller (and slightly less efficient) than the natural mechanism outcome.

B Public Bads

In this appendix we rewrite the problem as a public bads extraction problem. This formulation is very important, as public bads are related to some of the most important issues currently facing our planet such as global warming and over-fishing. As we discuss, this mechanism has the potential to provide a great deal of value in this domain, although some major public bads present unique challenges that this mechanism alone will not be sufficient to overcome.

Consider a public bads version of the game where

$$u_i = b_i - \delta_i g(B)$$

Here b_i is the amount extracted by player i and $B = \sum b_i$

Now $g(B)$ is a convex function

B.1 VCM Problem

As a point of reference, we first consider the public bads contribution problem when no mechanisms are in place. The amount players ask is equal to the amount they extract from the public good. In this VCM, each player is picking

$$b_i^* \in \arg\max_{b_i} b_i - \delta_i g(B)$$

This will implement

$$B_N^* \in \arg\max_{B} B - \min_{i}(\delta_i) * g(B)$$

Since the player with the lowest δ_i will keep extracting until this level is met. Note this is in some ways worse than the public goods example, because public bad levels are determined by the worst single player for the job rather than the best. If a player is not impacted at all by a public bad ($\delta_i = 0$), then the amount of public bad generated will essentially be infinite.

B.2 Planner Problem and Efficiency

The planner wants to maximize the sum of utilities. In other words they want to choose

$$B^* \in \arg\max_{B} B - g(B)$$

Because the payoffs are linear in contributions, the planner does not care how the extractions are divided among participants. By inspection, we can see that the VCM will not produce the efficient sum of total extractions.

B.3 Optimal Ask Mechanisms

The core features of the MCM is that all players pay the same adjusted amount in and that palyers are locally solving the planning problem. In principal this could be based on the maximum contribution or quantile contributions without changing much. The minimum contribution was chosen to ensure that players do not pay in more than they offer and to guarantee individual rationality for participation in the mechanism. The outside option of paying nothing into the mechanism is fairly obvious.

With public bads, the outside option is a bit less obvious. Is the outside option extracting nothing or extracting as much as you want? I suggest two possible mechanisms based on whether it is easier to prevent extraction or force extraction of the public bad. If neither one is possible, then the mechanism is unlikely to be successful in any form

Minimum Ask

If extraction can be easily prevented, the outside option would be all players extracting nothing. Here a minimum ask mechnism can be used. Each player asks for an extraction level x_j. They are allowed to extract.

$$b_i = \delta_i * \min_j \tfrac{x_j}{\delta_j}$$

Note that in this case

$$B = \sum_{i=1}^{I} b_i = \sum_{i=1}^{I} \delta_i * \min_j \tfrac{x_j}{\delta_j} = \min_j \tfrac{x_j}{\delta_j}$$

Under this mechanism, players extract a weighted amount based on the lowest adjusted proposed extraction.

Note that in this setup the player i picks x_i to solve

$$x_i^* \in \arg\max_{x_i} \delta_i * g\left(\min\left(\tfrac{x_i}{\delta_i}, \tfrac{x_k}{\delta_k}\right)\right) - \min\left(x_i, \delta_i \tfrac{x_k}{\delta_k}\right)$$

where

$$k = \arg\min_{j \neq i} \tfrac{x_j}{\delta_j}$$

As in the public goods case, this mechanism implements the efficient outcome in dominant strategies. It is also individually rational to participate in this mechanism, since non-participation has the same effect as asking for zero extraction, which is an option.

In cases where the public bad is finite, and it is efficient to extract it all, one can simply split the quantity based on δ_i

Maximum Ask

The case where the public bad is not excludable but extraction can be enforced, a slightly different mechanism is needed. In this case, a player may choose to forego participation in the mechanism and extract as much as they want. In this case, one could use a maximum ask mechanism which forces each player to extract.

$$b_i = \delta_i * \max_j \tfrac{x_j}{\delta_j}$$

In this case an extraction by a non-participant can be treated as an ask. This mechanism also implements the efficient extraction level in dominant strategies, but it brings a number of unique challenges.

First if the public bad is finite, it may not be possible to force extraction level dictated by the mechanism.

There is also a serious fairness concern in cases where historical extraction has already taken place. One could theoretically treat historical extraction as part of the ask, but that might lead to catastrophic extraction levels, since those extractions were not made with this mechanism in mind. In that case some kind of start date for the mechanism must be chosen. Earlier dates are generally fairer in an absolute sense, but they are also likely to lead to inefficiently high extraction.

In general the case where extraction of the public bad cannot be blocked are challenging.

C Checked Leader MCM

The MCM as described requires the mechanism maker to have full knowledge of the preferences of all individuals involved in the public good as is typical in discussions of the PPM. However, there are other informational environments that the PPM can be adapted to. For example, the PQ mechanism of Van Essen and Walker (2017) works well in environments where the mechanism maker has no knowledge but preferences are common knowledge among participants.

We can also adapt the MCM for such an environment. There are actually a number of ways to do this, but we will focus on the most illustrative called Checked Leader MCM (CL-MCM).

This mechanism enriches the MCM into a 3 stage game. There are two players (a Leader and a Checker) who each have special roles in this mechanism, they should be selected based on the amount of information they possess. Say that Player 1 is the Checker and Player 2 is the Leader

The Checked Leader MCM goes like this

1. The Checker picks p_2 for the Leader
2. The Leader picks $\boldsymbol{p}_{-2}$ (all p_i other than p_2) subject to $\sum_{i \neq 2} p_i = 1 - p_2$
3. Players play MCM game

After the final stage, the checker faces a penalty if $Y_2 \neq Y_3$. This guarantees p_2 is picked correctly, since any other pick will cause discrepancy. Note that this penalty does remove the guarantee of budget balance and of individual rationality for the checker, although this will not be relevant in equilibrium and the mechanism can be guaranteed not to run a deficit. In practice the other participants could provide a prize to the checker with the penalty being not receiving the prize. This could restore the guarantee of individual rationality.

For this game we introduce a new equilibrium refinement.

Definition 1. A Weak Dominant Strategy Subgame Perfect Equilibrium (WDSSPE) is a subgame perfect equilibrium where players play weakly dominant strategies in any subgame where they are available.

Given this definition we have the following result

Proposition 8. *Assuming common knowledge of preferences with the group and that all utility functions are continuously twice differentiable and strictly concave, then there exists a WDSSPE of the CL-MCM implements the Lindahl Equilibrium and reveals Lindahl Prices.*

Proof. Stage 3: We work by backwards induction. We know in the final stage that each person has one weakly dominant action (strict concavity gives uniqueness), and that each players dominant actions are decreasing in p_i. Call the resulting adjusted offer $Y_i(p_i)$. Note that changing one's adjusted offer within $Y_i(p_i)$ will not influence the player's expected payoff by definition.

Stage 2: Player 2's selection of $\boldsymbol{p}_{-2}$ only influences his payoff through the restriction it places on his maximum effective adjusted offer. This restriction is $\bar{Y} = \min_{i \neq 2} Y_i(p_i)$. By continuity of the second derivatives of utility, this restriction is maximized $Y_i(p_i) = Y_j(p_j) \forall i, j \neq 2$. Call this number $\bar{Y}(p_2)$. Note that $\bar{Y}(p_2)$ is strictly decreasing in $\bar{Y}(p_2)$. Assume that Player 2 will always pick a $\boldsymbol{p}_{-2}$ that achieves $\bar{Y}(p_2)$

Note that when $p_2 = p_2^*$ this restriction will be binding as long as $\bar{Y} < Y^*$. Furthermore, the only way to achieve $\bar{Y} = Y^*$ in this case is to set $p_i = p_i^* \forall i \neq 2$.

Stage 1: Player 1 may have some incentive to set p_2 higher than p_2^* ordinarily (in order to slightly reduce p_1) but this incentive can be counteracted by the arbitrarily large mismatch penalty. Note that in equilibrium $Y_3/p_3 = \bar{Y}(p_2)$ which is strictly decreasing in p_2. Furthermore $Y_2(p_2)$ is strictly increasing in p_2, and $\bar{Y}(p_2^*) = Y_2(p_2^*)$. Therefore, the only way to avoid the penalty is to set $p_2 = p_2^*$

□

This proposition takes advantage of two facts. First that players wish to reveal their information about the preferences of others. Second, that when prices are correct, adjusted contributions match. This is not the only mechanism which can be used to elicit Lindahl Prices when they are common knowledge among participants. For example, one could use a mechanism where each player picks the price ratio for another pair of players and then pays a penalty for mismatch of their adjusted offers. This version, however, has potentially worse violations of budget balance.

One could also use a mechanism where each player proposes a price vector and then the true price vector is equal to the most frequently proposed vector. If no two players agree, then the MCM is not executed. This mechanism maintains the guaranteed budget balance and individual rationality, but it pays a cost in a much greater multiplicity of equilibria.

C.1 Information Extraction Example

In general, it is difficult to get full revelation due to the incentive player's have to lower their own price. It is sometimes even difficult to extract information players have about the preferences of others since it will generally impact their own price indirectly. However, when the efficiency motive dominates and the incentive for manipulation is low, it can be possible to get players to truthfully reveal their own prices.

Consider an environment where each player knows their own δ_i where $\delta_i \in \{\delta^L, \delta^M, \delta^H\}$ where $\delta^L < \delta^M < \delta^H$. Each player send the mechanism maker a signal $s_i \in \{L, M, H\}$. The mechanism makers then selects a vector of prices $\boldsymbol{p}$ summing to 1 as a function of messaged received. We are going to want the mapping to have two properties

Assume that any assignment of specific individuals to δs is equally possible a priori. Say that each player receives $s_i \in \{1,2,3\}\backslash\{i\}$ which is equal to j if $\delta_j > \delta_k$ with probability $\rho > 0.5$. In other words it is an informative signal about which other player has a higher δ.

$$g(Y) = \begin{cases} Y(1+\frac{1}{k}) & Y \leq k \\ k+1 & Y > k \end{cases}$$

Note under this public good production technology, if

$$\tfrac{p_i}{\delta_i} \leq 1 + \tfrac{1}{k}$$

Then $Y_i = Y^*$, otherwise $Y_i = 0$. We want to choose k large enough to guarantee that the mechanism designer cannot guess the order of δ wrong and still produce a payoff in order to simplify the discussion and avoid hedging. This means we want k large enough that there does not exist prices $p^L \leq p^M \leq p^H$ such that $\min Y_i > 0$ when p^j is not assigned to the player with δ^j where $j \in \{L, M, H\}$.

Lemma 1. *We can guarantee that $Y = 0$ in the matching prices game whenever whenever p^i is assigned to a player with δ^j if*

$$k > \max\left(\tfrac{\delta^L + 2\delta^M}{1-\delta^L-2\delta^M}, \tfrac{\delta^H+2\delta^L}{1-\delta^H-2\delta^L}\right)$$

Proof. Case 1 switching p^H and p^M
Maximum room to work with $p^L = \delta^L + \frac{\delta^L}{k}$
Say then that $p^M = 1 - p^H - \delta^L - \frac{\delta^L}{k}$
Minimum Y_i will be player Ms
Maximized when $p^H = p^M$
$p^H = 1 - p^H - \delta^L - \frac{\delta^L}{k}$
$p^H = \frac{1-\delta^L-\frac{\delta^L}{k}}{2}$
So to assure this produces $\min Y_i = 0$ need
$\frac{1-\delta^L-\frac{\delta^L}{k}}{2} \geq \delta^M + \frac{\delta^M}{k}$
$1 - \delta^L - \frac{\delta^L}{k} \geq 2\delta^M + \frac{2\delta^M}{k}$
$\left(1-\delta^L-2\delta^M\right)k \geq \delta^L + 2\delta^M$
$k \geq \frac{\delta^L+2\delta^M}{1-\delta^L-2\delta^M}$
Case 2 switching p^M and p^L
Maximum room to work with $p^H = \delta^H + \frac{\delta^H}{k}$
Say then that $p^L = 1 - p^M - \delta^H - \frac{\delta^H}{k}$
Minimum Y_i will be player Ls
Maximized when $p^M = p^L$
$p^M = 1 - p^M - \delta^H - \frac{\delta^H}{k}$
$p^M = \frac{1-\delta^H-\frac{\delta^H}{k}}{2}$

So to assure this produces $\min Y_i = 0$ need
$\frac{1-\delta^H-\frac{\delta^H}{k}}{2} \geq \delta^L + \frac{\delta^L}{k}$
$1 - \delta^H - \frac{\delta^H}{k} \geq 2\delta^L + \frac{2\delta^L}{k}$
$\left(1 - \delta^H - 2\delta^L\right) k \geq \delta^H + 2\delta^L$
$k \geq \frac{\delta^H+2\delta^L}{1-\delta^H-2\delta^L}$
Note that these conditions also imply that $\min Y_i = 0$ if p^H and p^L are switched
So we need

$$k > \max\left(\frac{\delta^L+2\delta^M}{1-\delta^L-2\delta^M}, \frac{\delta^H+2\delta^L}{1-\delta^H-2\delta^L}\right)$$

We can now present the following corollary

Proposition 9. *There is an equilibrium in the price matching prices setting in weakly dominant strategies where each player reveals δ_i truthfully.*

Proof. First we must formalize the mechanism used by the mechanism maker. We will assume that the use a simple and natural mechanism. If a player is the only one to send signal $s_i = j$ then they receive $p_i = \delta^j$. If multiple players send the same signal, then one is chosen randomly to receive the corresponding price and the other receives the remaining price. If all players send the same signal, then all receive p_i equal to a random δ^j.

We know that the player gets one of two payoffs: (1) the payoff that they get when each player gets the correct price and (2) the default payoff that they get when at least one player gets the wrong price. We also know (1) > (2). Therefore, the best action is the one which maximizes the chance of (1).

Case 1: All other players reveal truthfully. Player i revealing truthfully generates (1)with certainty.

Case 2: One other player sends an incorrect signal s_j. Sending the correct s_i generates (1) with probability 1/2 regardless of which wrong signal was sent. Sending a wrong signal not equal to s_j guarantees (2) since there is now a wrong signal uncontested by a right signal. Sending a wrong signal equal to s_j will lead to all players sending the same signal which means (1) is generated with a probability of 1/6.

Case 3: Both other players send an incorrect signal. There will always be an uncontested incorrect signal, so (2) is guaranteed.

Therefore, reporting truthfully is a weakly dominant strategy

□